Name:_____

Periods: (circle one) 1/2 3/4

INVERSE VARIATION

Introduction

In the "Bridge over Troubled Waters" investigations, you explored the relationship of strength, number of layers, and length of a bridge. You found that the relationship between strength and number of layers was approximately linear. You also found that the relationship between strength and length was not linear. In these investigations, you will explore other nonlinear functions.

Investigation 3-1

TASK 1

In recent years, the populations of many small towns have declined as residents move to large cities for jobs. The town of Roseville has a plan to attract new residents. Roseville offers free land to "homesteaders" who are willing to build houses. Each lot is rectangular and has an area of 21,780 square feet. The lengths and widths of the lots vary. The town planners want a quick way to check lot sizes for the new homesteaders.

What function relates the length and width of rectangles with area 21,780 square feet?

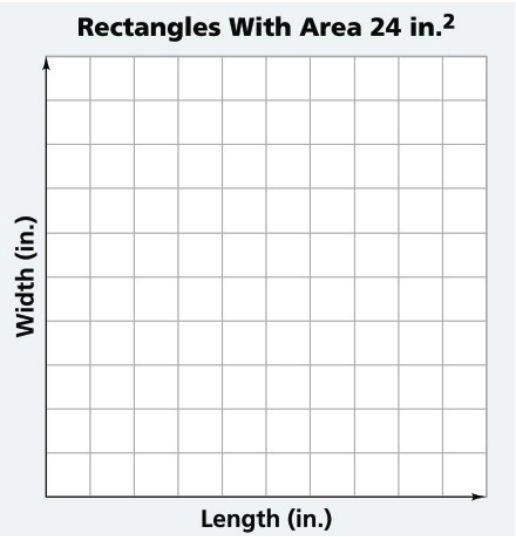
What patterns appear in tables and graphs of that function?

Task 2

Complete the table below. Then plot your (length, width) data from the table on a graph to the right. Then draw a line or curve that models the pattern in the data.

Rectangles With Area 24 in.^{2}

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)								



Name:_____

Periods: (circle one) 1/2 3/4

Describe the pattern of change in the width of a rectangle as the length increases. Is the relationship between length and width linear?

Write an equation relating width "w" to length "l" for rectangles with an area of 24 in^2.

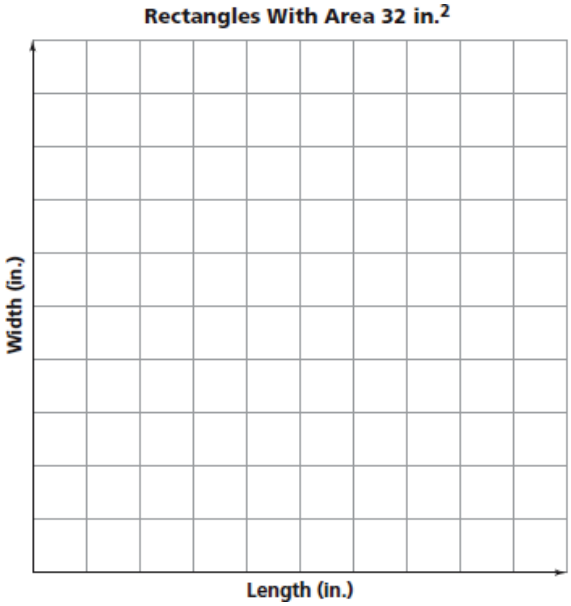
TASK 3

Now consider rectangles with area of 32 square inches.

Write an equation for the relationship between the width "w" and the length "l".

Complete the table and a graph of values for "w" and "l".

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)								



TASK 4

Compare tables, graphs, and rules for the functions relating width and length of rectangles with area 24 in^2 and area 32 in^2. Then use your results to answer the Roseville planners' questions about lots with area 21,780 square feet.

Name:_____

Periods: (circle one) 1/2 3/4

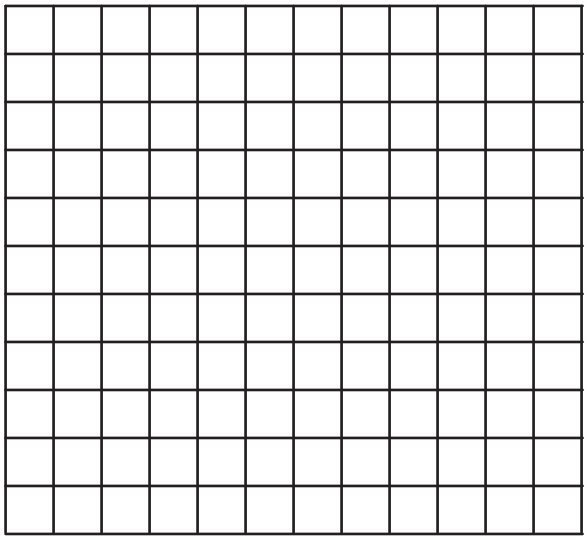
Applications 3-1

Consider rectangles with area of 16 square inches.

1) Complete the table and a graph of values for "w" and "l".

Rectangles With Area 16 in.^{2}

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)								



2) Describe the pattern of change in the width of a rectangle as the length increases.

3) Write an equation that shows how width "w" depends on the length "l." Is the relationship linear?

INVERSE VARIATION

Name:_____

Periods: (circle one) 1/2 3/4

Investigation 3-2

TASK 1

The relationship between length and width for rectangles with a fixed area is not linear. As "l" increases, "w" decreases, but not at a constant rate. The relationship between "l" and "w" is an example of an important relationship called an **inverse variation**. Two nonzero variables x and y are related by an inverse variation if

$$y = \frac{k}{x} \quad \text{or} \quad xy = k$$

where k is a constant other than 0.

How does the value of one variable change as the value of the other changes?

How is that pattern of change shown in a table of data and on a graph?

What equation shows how the two variables are related?

Name:_____
Periods: (circle one) 1/2 3/4

Investigation 3-2

TASK 2

Inverse variation occurs in many situations. You have probably thought about how the length of a trip depends on speed.

Let's take a look at Mr. Cordova. He lives in Detroit, Michigan and often travels to Baltimore, Maryland, to visit his grandfather. The trip is about 500 miles each way.

Here are Mr. Cordova's notes for his trips to Baltimore last year.

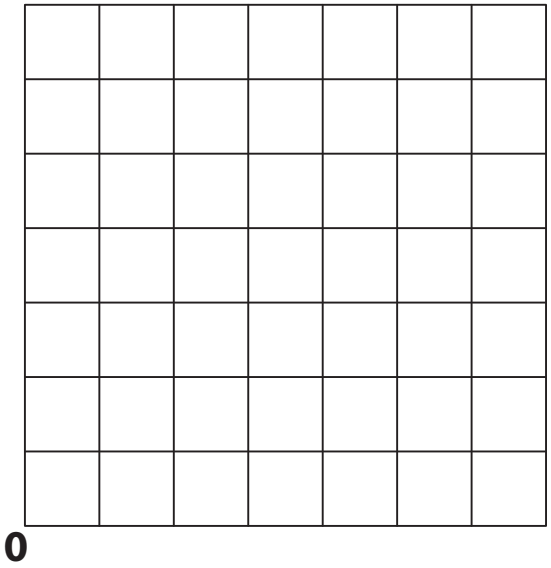
Date	Notes	Travel Time
February 15	Traveled by plane.	1.5 hours
May 22	Drove.	10 hours
July 3	Drove. Stopped for repairs.	14 hours
November 23	Flew. Flight delayed.	4 hours
December 23	Took overnight train.	18 hours

Calculate the average speed in miles per hour for each trip. Record your results the table below.

Mr. Cordova's Baltimore Trips

Travel Time (h)					
Average Speed (mi/h)					

Plot the data on a graph. Draw a line or curve to model the data. Describe the change in average speed as travel time increases.



Name:_____
Periods: (circle one) 1/2 3/4

Investigation 3-2

Write an equation for the relationship between time t and speeds.

Is the relationship between distance and time an inverse variation? Explain why or why not.

TASK 3

The Cordova family is planning a trip of 300 miles to Mackinac Island, near the upper peninsula of Michigan. Mr. Cordova does some calculations to see how the travel time will change if the family drives at different average speeds.

Travel Times for Different Speeds

Average Speed (mi/h)	30	40	50	60	70
Travel Time (h)	10	7.5	6	5	4.3

Describe the change in travel time as the average speed increases.

What equation relates travel time "t" to average speed "s"?

How is the pattern relating travel time to average speed shown in a graph of(s, t) data?

Is the relationship between travel time and average speed an inverse variation? Explain why or why not.

Name:_____

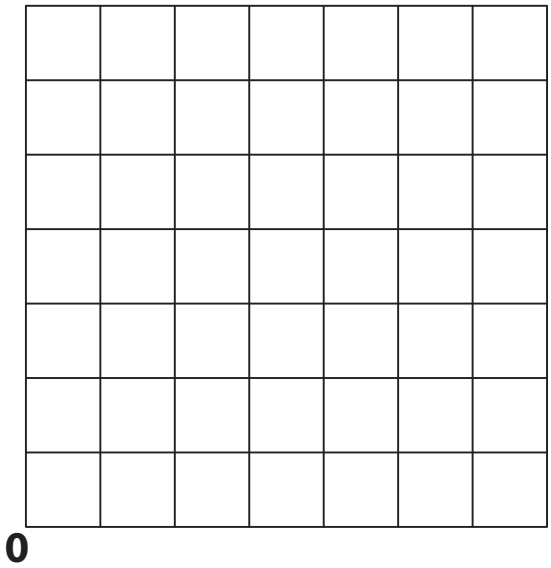
Periods: (circle one) 1/2 3/4

Investigation 3-2

TASK 4

Suppose Mr. Cordova decides to aim for an average speed of 50 miles per hour for the trip to Mackinac Island.

Make a table and a graph to show how the distance traveled will increase as time passes. Show times from when the family leaves home to when they reach their destination.



Describe the pattern of change in distance as time passes. Explain how that pattern is shown by values in your table and points on your graph.

Write an equation relating distance traveled “d” to time “t”.

Is the equation relating distance and time an inverse variation? Explain why or why not.

Name:_____

Periods: (circle one) 1/2 3/4

Applications 3-2

For the tables below, tell whether the relation between x and y is an inverse variation.

If it is, write an equation for the relationship.

x	1	2	3	4	5	6	7	8	9	10
y	10	9	8	7	6	5	4	3	2	1

x	1	2	3	4	5	6	7	8	9	10
y	48	24	16	12	9.6	8	6.8	6	5.3	4.8

x	2	3	5	8	10	15	20	25	30	40
y	50	33	20	12.5	10	6.7	5	4	3.3	2.5

x	0	1	2	3	4	5	6	7	8	9
y	100	81	64	49	36	25	16	9	4	1

Name:_____

Periods: (circle one) 1/2 3/4

Investigation 3-3

TASK 1

The science teachers at Everett Middle School want to take their eighth-graders on a field trip to a nature center. It costs $750 to rent the center facilities.

The school budget does not provide funds to rent the nature center, so students must pay a fee. The trip will cost $3 per student if all 250 go. The teachers know, however, that it is unlikely that all students will go. They want a way to find the cost per student for any number of students.

What kind of relationship between number of students and cost should the teachers expect?

How can that relationship be expressed with an equation and a graph?

TASK 2

To identify the relationship between the number of students and the cost, begin with sample calculations. Then look for a pattern in your results.

Complete the table below

Number of Students	25	50	75	100	125	150	175	200	250
Cost per Student (dollars)									

Describe the pattern relating the cost per student to the number of students who visit the nature center.

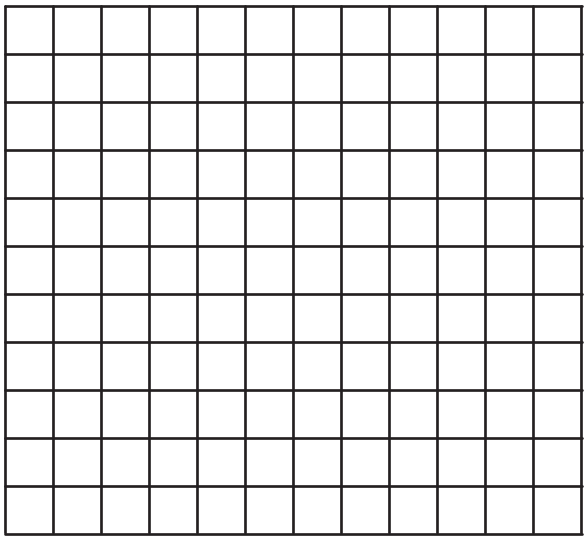
Name:_____

Periods: (circle one) 1/2 3/4

Investigation 3-3

Write an equation relating cost per student "c" to number of students "n."

Sketch a graph showing how the cost per student changes as the number of students increases.



TASK 3

Find the change in the cost per student as the number of students increases in the following ways:

from 10 to 20? from 100 to 110? from 200 to 210?

Is the function relating number of students and cost linear? Explain.

Do equal increases in numbers of students cause equal decreases in cost per student?

Name:_____
Periods: (circle one) 1/2 3/4

Investigation 3-3

TASK 4

How will doubling the number of students change the cost per student? To test your ideas about that question, find answers to these related questions.

Find the change in per-student cost as the number of students increases in the following ways:

from 20 to 40? from 40 to 80? from 80 to 160?

Describe any patterns you see in your answers above.

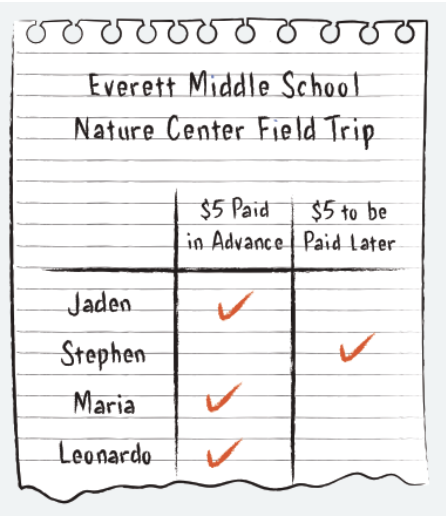
How does your equation from Task 2 help to explain the effect of doubling the number of students?

The science teachers decide to charge $5 per student for the field trip. They will use any extra money to buy science equipment for the school.

Write an equation for the amount "A" the teachers will collect if n students go on the trip.

2. Sketch a graph of the relationship.

3. Does the graph show a linear relationship or an inverse variation? Explain.



Name:_____

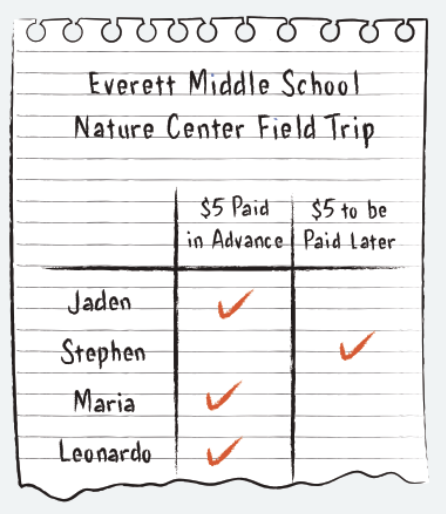
Periods: (circle one) 1/2 3/4

Investigation 3-3

TASK 5

The science teachers decide to charge $5 per student for the field trip. They will use any extra money to buy science equipment for the school.

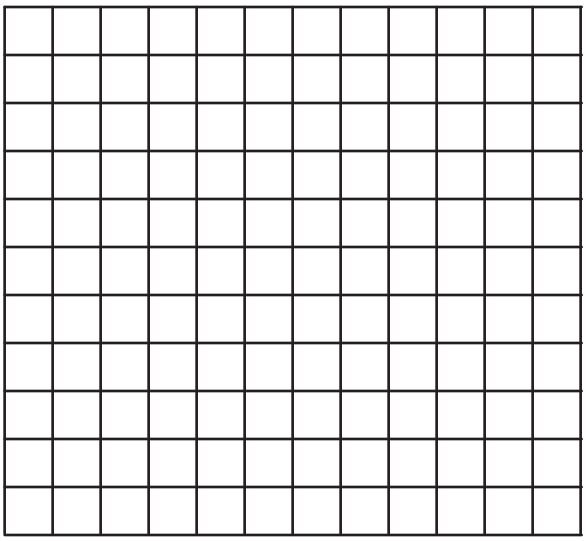
Write an equation for the amount "A" the teachers will collect if "n" students go on the trip.



Everett Middle School
Nature Center Field Trip

	$5 Paid in Advance	$5 to be Paid Later
Jaden	✓	
Stephen		✓
Maria	✓	
Leonardo	✓	

Sketch a graph of the relationship.



Does the graph show a linear relationship or an inverse variation? Explain.

Name:_____

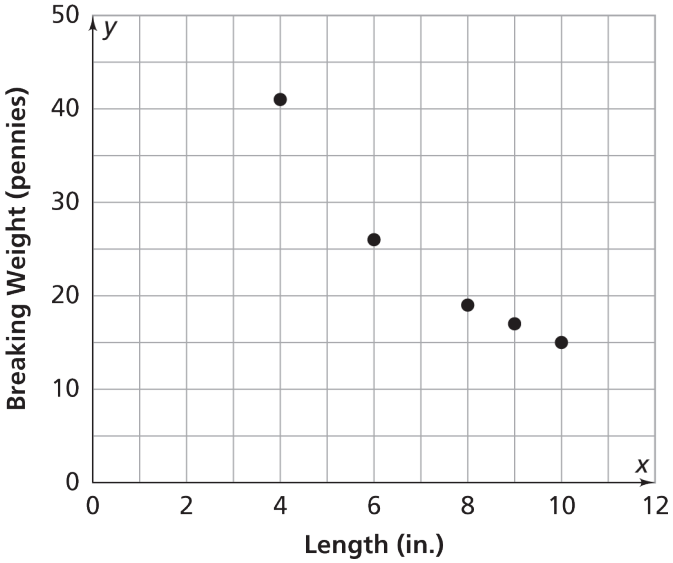
Periods: (circle one) 1/2 3/4

Investigation 3-4

TASK 1

In many real-world problems it is impossible to find an equation that fits given data exactly. For example, consider the table and graph below. They show the bridge experiment data collected by a group of students.

Length (in.)	Breaking Weight (pennies)
4	41
6	26
8	19
9	17
10	15



Do the data suggest the relationship between length and breaking weight is linear or an inverse variation?

What equation would model the relationship well?

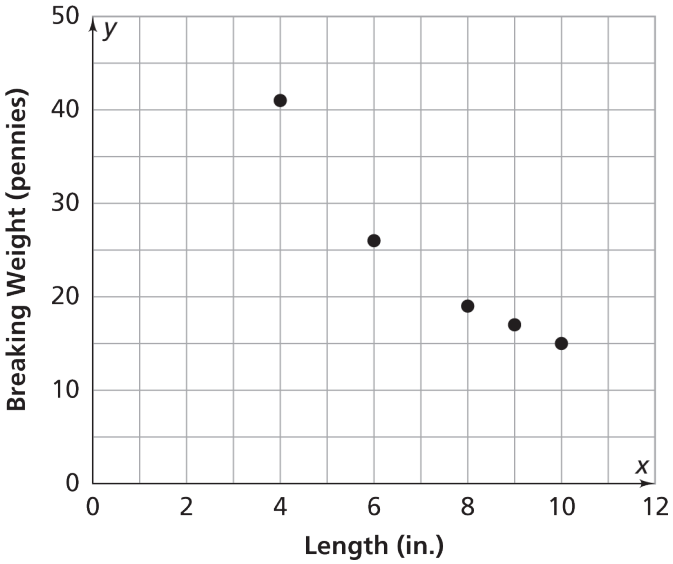
What do you see in the table and the graph that suggests an inverse variation relationship between breaking weight "w" and bridge length "l"?

Name:_____

Periods: (circle one) 1/2 3/4

Investigation 3-4

Length (in.)	Breaking Weight (pennies)
4	41
6	26
8	19
9	17
10	15



What value of k would make the equation k = l*w true for a length of 4 inches?

For a length of 6 inches?

What equation is a good model for the function relating weight "w" to length "l" ?

What breaking weights does your model predict for bridges of length 3, 5, 7, & 11 in.?

Name:_____
Periods: (circle one) 1/2 3/4

Applications 3-4

A student collected these data from the bridge-length experiment.

Bridge-Length Experiment

Length (in.)	4	6	8	9	10
Breaking Weight (pennies)	24	16	13	11	9

Find an inverse variation equation that models the data.

Explain how your equation shows that breaking weight decreases as length increases. Is this decrease reasonable for the situation? Explain.